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MODELING OF THERMAL STRAINS OF MULTILAYER SHELL-TYPE SPACE STRUCTURES

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A procedure of modeling of the stressed-strained state of large-size space structures is considered. Using the mirror of the concentrator of a solar power plant as an example, results of the modeling are shown.

Stringent requirements on the stability of shape in the process of operation are imposed on modern high-precision large-size space structures (antennas, concentrators, reflectors, etc.). Since the experimental development of such products under terrestrial conditions necessitates the creation of sophisticated and expensive equipment, it becomes more pressing to develop procedures of theoretical investigation of the stressed and strained state that enable one to take into account the distinctive features of the physical properties of the structural materials used.

In [1], the regimes of temperature loading which are characteristic of large-size space structures of composite materials that are in near-earth orbits have been analyzed and the critical moments of attainment of the maximum and minimum temperature levels have been determined; also, the temperature state of the structure for the selected calculated cases has been determined. This work seeks to investigate the strained state of a composite structure with a prescribed temperature field.

Problems of straining of isotropic thin-walled structures under the action of a change in the temperature gradients have been addressed for a long time already [2–5], and the corresponding range of problems has been studied in sufficient detail. However, the problem of calculation is substantially complicated for composite anisotropic materials in view of the distinctive features of their structure [6].

In particular, one characteristic feature of carbon-filled plastics is a pronounced dependence of the mechanical properties on the temperature [7]. The influence of the temperature dependence of the temperature coefficient of linear expansion on the strained state of a carbon-filled-plastic structure has been considered in [8] with the example of the simplest problem. It was shown that in three-layer structures with load-carrying layers of carbon-filled plastic for the same value of the temperature difference between the casings not only can the change in the curvature take on different values but it can change its sign as well. Therefore, evaluation of the strained state without taking into account the behavior of the material can lead to erroneous results.

In this work, we propose a procedure for determination of the stressed-strained state of mildly sloping composite shell-type structures with a prescribed temperature field; this procedure is based on the finite-element method and makes it possible to take into account the dependence of the elastic characteristics and the temperature coefficient of linear expansion of materials on the temperature.

Strain Relations. The material of the shells will be considered to be incompressible in the transverse direction. When the elasticity modulus is $E_z \to \infty$ the strain of transverse compression $\varepsilon_{3(z)}$ can be taken to be equal to zero. Then the normal displacement of a shell ω remains constant along the coordinate z, i.e.,

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$$v_3(z, x_1, x_2) = w(x_1, x_2)\,. \tag{1}$$

We take the middle surface of a shell as the coordinate surface. The coordinate surface ($z = 0$) of a mildly sloping shell is prescribed in the form $x_3 = x_3(x_1, x_2)$. Here x_1 and x_2 are considered to be the coordinates of a point on the projection plane of the mildly sloping shell, while x_3 is considered to be the coordinate along the third axis of a Cartesian coordinate system.

The assumption of the incompressibility of the shell in the transverse direction can be considered as the first kinematic hypothesis of the theory.

As the second kinematic hypothesis we adopt the assumption of a linear distribution of the tangential displacements v_1 and v_2 over the shell thickness, i.e.,

$$v_1(z, x_1, x_2) = u_1(x_1, x_2) + z\theta_1(x_1, x_2)\,. \tag{2}$$

Based on the classical definitions of the linear components of the strain tensor and on the assumptions of the mildly sloping nature of the shell, we derive the strain relations

$$\boldsymbol{\varepsilon} = \mathbf{L}\mathbf{u}\,, \tag{3}$$

or in expanded form

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \\ \kappa_1 \\ \kappa_2 \\ \chi_{12} \\ \gamma_{13} \\ \gamma_{23} \end{bmatrix} = \begin{bmatrix} \nabla_1 & 0 & -x_{3,11} & 0 & 0 \\ 0 & \nabla_2 & -x_{3,22} & 0 & 0 \\ \nabla_2 & \nabla_1 & -2x_{3,12} & 0 & 0 \\ 0 & 0 & 0 & \nabla_1 & 0 \\ 0 & 0 & 0 & 0 & \nabla_2 \\ 0 & 0 & 0 & \nabla_2 & \nabla_1 \\ x_{3,11} & x_{3,12} & \nabla_1 & 1 & 0 \\ x_{3,12} & x_{3,22} & \nabla_2 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \\ w \\ \theta_1 \\ \theta_2 \end{bmatrix}. \tag{4}$$

Here $\nabla_1 = \partial/\partial x_1 (1, 2)$ and $x_{3,ij} = \partial^2 x_3/(\partial x_i \partial x_j)$ ($i, j = 1, 2$). The indices after the comma mean differentiation with respect to the corresponding coordinate. For the case where the directions of the ox_1 and ox_2 axes coincide with the directions of the lines of principal curvatures, $x_{3,12} = 0$.

Thermoelasticity Relations. Thermoelasticity relations are considered under the assumption that the material of the structure is orthotropic and incompressible in the transverse direction. In this case the number of independent coefficients of elasticity is equal to six. With account for the thermal strains these relations can conveniently be written in the coordinate system of an individual layer (the index of the layer is omitted):

$$\boldsymbol{\sigma}' = \mathbf{C}'_\varepsilon \boldsymbol{\varepsilon}' - \boldsymbol{\sigma}'_T\,; \quad \boldsymbol{\tau}' = \mathbf{C}'_\gamma \boldsymbol{\gamma}'\,, \tag{5}$$

where $\boldsymbol{\sigma}' = [\sigma'_1, \sigma'_2, \sigma'_{12}]^T$, $\boldsymbol{\tau}' = [\sigma'_{31}, \sigma'_{32}]^T$, $\boldsymbol{\varepsilon}' = [\varepsilon'_1, \varepsilon'_2, \varepsilon'_{12}]^T$, and $\boldsymbol{\gamma}' = [\varepsilon'_{31}, \varepsilon'_{32}]^T$ are the column vectors of stresses and strains determined in the coordinate system of the layer; $\sigma_T = [\sigma'_{1T}, \sigma'_{2T}, 0]^T$ are the temperature components of the stresses;

$$\mathbf{C}'_\varepsilon = \begin{bmatrix} c'_{11} & c'_{12} & 0 \\ c'_{12} & c'_{22} & 0 \\ 0 & 0 & c'_{33} \end{bmatrix}; \quad \mathbf{C}'_\gamma = \begin{bmatrix} c'_{13} & 0 \\ 0 & c'_{23} \end{bmatrix};$$

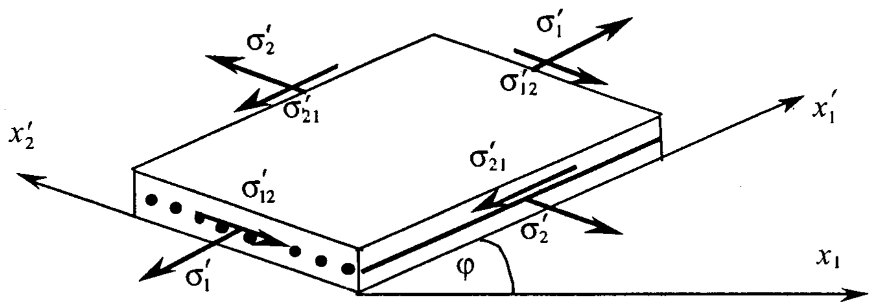


Fig. 1. Stresses in the element of a monolayer.

$$c'_{11} = \frac{E_1}{1 - \nu_{12}\nu_{21}} \quad (1, 2)\,; \quad c'_{12} = c'_{11}\nu_{21}\,; \quad c'_{33} = G_{12}\,; \quad \sigma'_{1T} = E_1\,(\alpha_1^0 + \nu_{21}\alpha_2^0)\,\Delta T/(1 - \nu_{12}\nu_{21}) \quad (1, 2)\,.$$

Relations (5) in the coordinate system tied to the shell (x_1, x_2, x_3) will have the form

$$\boldsymbol{\sigma} = \mathbf{C}_\varepsilon \boldsymbol{\varepsilon} - \boldsymbol{\sigma}_T\,; \quad \boldsymbol{\tau} = \mathbf{C}_\gamma \boldsymbol{\gamma}\,, \tag{5*}$$

where $\mathbf{C}_\varepsilon = \beta_\varepsilon^T \mathbf{C}'_\varepsilon \beta_\varepsilon$ and $\mathbf{C}_\gamma = \beta_\gamma^T \mathbf{C}'_\gamma \beta_\gamma$; β_ε and β_γ are the transformation matrices of the components of the column vector of strains in passage from the coordinate system of the layer to the coordinate system of the shell [6].

Since consideration is given to the thermal loading of the structure, of greatest interest are the temperature components of stresses which for an orthotropic layered material will be calculated as follows [6]:

$$\begin{aligned}
\sigma_{1T} &= (\cos^2\varphi\,(c'_{11}\alpha_1^0 + c'_{12}\alpha_2^0) + \sin^2\varphi\,(c'_{12}\alpha_1^0 + c'_{22}\alpha_2^0))\,; \\
\sigma_{2T} &= (\sin^2\varphi\,(c'_{11}\alpha_1^0 + c'_{12}\alpha_2^0) + \cos^2\varphi\,(c'_{12}\alpha_1^0 + c'_{22}\alpha_2^0))\,; \\
\sigma_{12T} &= T\sin\varphi\cos\varphi\,(c'_{11}\alpha_1^0 + c'_{12}\alpha_2^0 - c'_{12}\alpha_1^0 - c'_{22}\alpha_2^0)\,,
\end{aligned} \tag{6}$$

where $c'_{11} = \dfrac{E_1}{1 - \nu_{12}\nu_{21}}$, $c'_{22} = \dfrac{E_2}{1 - \nu_{21}\nu_{12}}$, and $c'_{12} = c'_{11}\nu_{21}$ are the coefficients of elasticity of the material in the coordinate system of the layer (Fig. 1).

Internal Force Factors. In order to determine the strained state of a structure subject to thermal loading we determine the values of the temperature forces and moments occurring in a multiple sandwich.

For a multiple sandwich formed by cross packing of the layers the components of the temperature forces and moments are determined by integration of the corresponding components of thermal stresses over the sandwich thickness:

$$N_{1T} = \int_{H_\Sigma} \sigma_{1T}\,(1 + k_1 z)\,dz \quad (1, 2)\,; \quad M_{1T} = \int_{H_\Sigma} \sigma_{1T}\,z\,(1 + k_1 z)\,dz \quad (1, 2)\,, \quad H_\Sigma = H_0 + 2h_0\,. \tag{7}$$

Since we consider the symmetric scheme of packing and the temperature difference over the sandwich thickness is insignificant, as follows from the results of [1], the stresses σ_{12T} for the sandwich can be taken to be equal to zero. As a consequence, the corresponding components of the temperature forces and moments are $N_{12T} = M_{12T} = 0$.

Considering the thermal conductivities of the ith layer $\lambda_{[i]}$ to be constant within the layer, we can perform the integration (7). In calculating the integrals, we will assume that within the ith layer

$$1 + k_1 z \approx 1 + k_1 t_{[i]} \quad (1, 2)\ ; \quad \sigma_{1T}^{[i]} = d_{1T}^{[i]}\, T_{[i]} \quad (1, 2)\ ;$$

$$d_{1T}^{[i]} = \cos^2 \varphi_{[i]} \,(c_{11}^{[i]}\, \alpha_1^{0[i]} + c_{12}^{[i]}\, \alpha_2^{0[i]}) + \sin^2 \varphi_{[i]} \,(c_{12}^{[i]}\, \alpha_1^{0[i]} + c_{22}^{[i]}\, \alpha_2^{0[i]}) \quad (1, 2)\ .$$

As a result of integration we obtain the values of the temperature forces and moments for a multiple sandwich (Fig. 2):

$$N_{1T} = \sum_{i=1}^{n} d_{1T}^{[i]}\,(1 + k_2 t_{[i]})\, T_N^{[i]} \quad (1, 2)\ ; \quad M_{1T} = \sum_{i=1}^{n} d_{1T}^{[i]}\,(1 + k_2 t_{[i]})\, T_M^{[i]} \quad (1, 2)\ , \tag{8}$$

where

$$T_N^{[i]} = T_{(i-1)} t_1^{[i]} - q\,(t_2^{[i]} - z_{(i-1)} t_1^{[i]}) / \lambda_{[i]}\ ; \quad T_M^{[i]} = T_{(i-1)} t_2^{[i]} - q\,(t_3^{[i]} - z_{(i-1)} t_2^{[i]}) / \lambda_{[i]}\ ;$$

$$t_{[i]} = \frac{z_{(i-1)} + z_{(i)}}{2}\ ; \quad t_1^{[i]} = h_{[i]}\ ; \quad t_2^{[i]} = \frac{z_{(i)}^2 - z_{(i-1)}^2}{2}\ ; \quad t_3^{[i]} = \frac{z_{(i)}^3 - z_{(i-1)}^3}{3}\ ; \quad q = \frac{T_{\mathrm{d}} - T_{\mathrm{u}}}{\sum_n \dfrac{h_{[i]}}{\lambda_{[i]}}}\ .$$

Thus, formulas (7) and (8) determine the components of the temperature forces and moments occurring in the multilayer shell-type structure. Knowing the distributions of the temperatures T_{u} and T_{d} over the upper and lower sandwich surfaces from the thermal calculations, we can determine the internal force factors and next, using the physical relations of the theory of mildly sloping shells [6] and the equations of connection of strains with displacements, determine the strained state of the structure in question.

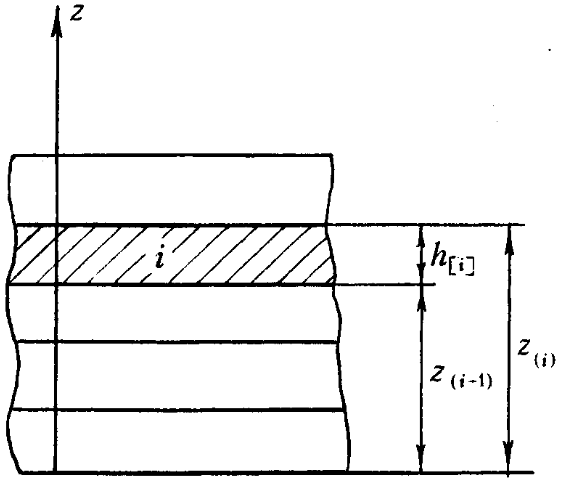
Finite-Element Model. To pass from the analytical dependences to a numerical solution one uses the procedures of the finite-element method. In [9], the authors proposed a new nine-nodal tetragonal finite element which uses the strain relations of a mildly sloping shell of a general form. Such a finite element makes it possible to carry out convenient subdivision of a shell-type structure, possesses good convergence, and enables us to take into account the characteristic features of composite materials in calculations, i.e., the nonuniformity of the structure in thickness, low rigidity in lateral shear, and the dependence of the elasticity moduli and of the temperature coefficient of linear expansion on the temperature. It should be noted that the strain relations based on which the element was constructed have been derived for the general case of a mildly sloping shell where the coordinate axes ox_1 and ox_2 do not coincide with the lines of principal curvatures. This provides the possibility of applying this element to calculation of a wider class of shell-type structures.

To approximate stiffeners one can use a three-nodal one-dimensional finite element [10].

Example. In this work, the problem formulated is solved with the example of the structure of the lobe of a parabolic reflector of a promising solar gas-turbine power plant; this plant is intended for use as the primary energy source in onboard systems of manned spacecraft [11].

It has been assumed that structurally, the reflector lobe is a mildly sloping stiffened multilayer shell which consists of two carbon-filled-plastic casings of thickness h_0 and a filler, i.e., aluminum honeycombs of height H_0. Each casing consists of four unidirectional layers of carbon-filled plastic packed according to the scheme $[0^{\mathrm{o}}/90^{\mathrm{o}}/90^{\mathrm{o}}/0^{\mathrm{o}}]$. The monolayer thickness was taken to be $h_{\mathrm{m}} = 0.01 H_0$.

In approximation of the shell surface, 320 two-dimensional finite elements were used. To approximate the stiffening beam and two longitudinal edgings we used 120 linear finite elements (40 per object). The transverse edging was approximated by eight finite elements. It was assumed that the reflector lobe was rapidly fixed to a narrow (internal) transverse edge; the remaining edges were considered to be free.



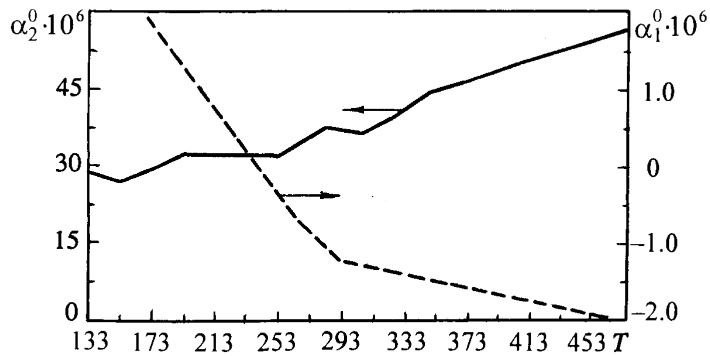


Fig. 2. Structure of a multiple sandwich.

Fig. 3. Model temperature dependences of α_1^0 and α_2^0 (1/K).

As has been noted earlier, a characteristic feature of carbon-filled plastics is a pronounced dependence of the mechanical properties on the temperature. Figure 3 gives plots of the change of the temperature coefficient of linear expansion of a carbon-filled plastic, used in the structure, in the temperature range 173–473 K. It is seen from the plots that the quantity α_1^0 undergoes the largest relative changes in the above temperature range; the sign of the temperature coefficient of linear expansion changes to a negative one (i.e., the monolayer is contracted in heating).

It has been shown in [6] that for shell-type structures the temperature dependence of the properties leads to the appearance of nonzero mixed (membrane-flexural) rigidity characteristics, which is a nonremovable feature in this case. In this connection, design calculations are substantially increasing in importance.

In the calculations carried out, the dependence of the elasticity moduli $E_1(T)$, $E_2(T)$, and $G_{12}(T)$ was taken into account at the stage of determination of the membrane, flexural, and mixed rigidity characteristics of the structure. The influence of the dependences of the temperature coefficient of linear expansion $\alpha_1^0(T)$ and $\alpha_2^0(T)$ was taken into account at the stage of determination of the temperature forces and moments, i.e., in calculation of the combination $d_{1T}^{[i]}$ (1, 2), we substitute into it the values of the temperature coefficient of linear expansion of the ith layer $\alpha_1^{0[i]}(T_{[i]})$ (1, 2) which are determined for the temperature of the ith layer $T_{[i]}$.

It was assumed that the structure of the reflector lobe is subject to only thermal loading; consideration was given to two basic calculated cases determined in [1]. The first of them corresponded to the moment of termination of the shadow portion of the orbit and hence the minimum levels of temperature in the members of the structure. The second case corresponded to the moment of termination of the illuminated portion of the orbit and to the maximum temperature levels. The values of the temperatures were prescribed for the upper and lower surfaces of the shell, the stiffening beam, and the edgings for three cross sections that corresponded to the longitudinal coordinate $x = 0$, $x = L/2$, and $x = L$ (L is the length of the reflector lobe). The temperatures in the intermediate cross sections were determined by interpolation from the prescribed three cross sections.

In analyzing the results of calculations of the strained state, the emphasis was on the values of the normal displacements of the shell surface since it is precisely they that exert a determining influence on the effectiveness of the structure. The calculations were carried out on a personal computer with a 100-MHz Pen-

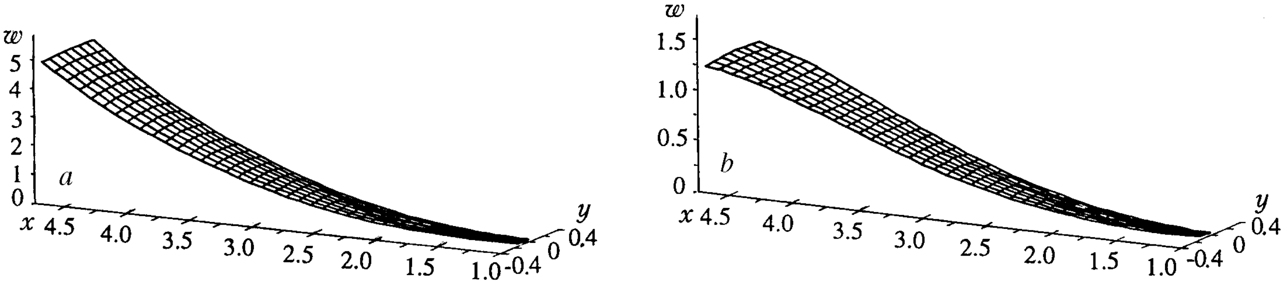


Fig. 4. Normal displacements of the shell surface for calculated cases 1 (a) and 2 (b). ω, mm; x and y, m.

tium I processor and an on-line storage of 32 Mb and took about 4 min. The calculation results are presented in Fig. 4 in the form of the diagrams of distribution of the normal displacements.

It is seen from the given results that the normal displacements attain the highest value in the first calculated case, i.e., for the minimum temperature level. This is explained by a significant relative decrease in the value of the temperature coefficient of linear expansion α_1^0 in heating and by its "nontraditional" negative value in the temperature range in question [7]. It should be noted that in both calculated cases, normal displacements of one sign (positive ones) occur.

As possible methods for decreasing the values of the normal displacements of the surface of a reflector and other shell-type structures operating under similar conditions we can recommend the following variants:

(a) shifting the range of workloads to the region of high temperatures by deposition of a coating with the corresponding characteristics of reflection and absorption;

(b) when the strain of the reflector was considered it was assumed that an ideal reflecting surface is realized at a temperature of 273 K relative to which the increments in the temperature are calculated for computation of strains. To decrease the values of the deviation from an ideal reflecting surface we can recommend that the reflector lobe be manufactured in such a way that this surface (or a surface close to it) would be realized in the region of decreased temperatures. However the above method requires that the technological aspects of the problem be studied in detail;

(c) a safe method for attaining the minimum deviations from an ideal reflecting surface is thermostatic control of the structure, which, however, will involve an increase in its mass.

CONCLUSIONS

1. The procedure of calculation of the stressed-strained state of multilayer stiffened mildly sloping shells with a prescribed temperature field has been developed and realized with account taken of the dependence of the elastic characteristics and the temperature coefficient of linear expansion on the temperature.

2. Within the framework of this procedure, a new tetragonal multilayer finite element of a mildly sloping shell of general form has been developed based on a mixed variational formulation.

3. With the use of the proposed procedure, the important problem of investigation of the geometric stability of the large-size structure of a parabolic mirror, i.e., the concentrator of a solar gas-turbine power plant, has been solved practically.

4. A package of applied programs has been developed for solution of this class of problems.

NOTATION

E_z, elasticity modulus in the transverse direction; x_1, x_2, x_3, Cartesian coordinate system tied to the shell; v_1, v_2, and v_3, displacements of a point on the shell surface along the coordinate lines; u_i (i = 1, 2), tangential displacement of a point of the coordinate surface along the ox_i axis in the process of straining; θ_i (i = 1, 2), angle of rotation of the cross section in the plane zox_i; ε_1, ε_2, and γ_{12}, membrane strains of the coordinate surface; κ_1, κ_2, and χ_{12}, changes in the curvatures; γ_{13} and γ_{23}, lateral-shear strains; $\boldsymbol{\varepsilon}$, vector of generalized strains; $\mathbf{u}$, vector of generalized displacements; $\mathbf{L}$, differential operator; x_1', x_2', x_3', coordinate system of the layer; E_1, E_2, G_{12}, G_{13}, and G_{23}, moduli of tension-compression and shearing of the layer in the coordinate system tied to the orthotropism axes; ν_{12} and ν_{21}, coefficients of transverse strains (Poisson coefficients); α_1^0 and α_2^0, temperature coefficients of linear expansion; ΔT, temperature change; $\boldsymbol{\sigma}'$ and $\boldsymbol{\tau}'$, column vectors of stresses determined in the coordinate system of the layer; $\boldsymbol{\varepsilon}'$ and $\boldsymbol{\gamma}'$, column vectors of strains determined in the coordinate system of the layer; $\boldsymbol{\sigma}_T'$, vector of temperature stresses in the coordinate system of the layer; $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$, column vectors of stresses determined in the coordinate system of the shell; $\boldsymbol{\varepsilon}$ and $\boldsymbol{\gamma}$, column vectors of strains determined in the coordinate system of the shell; $\boldsymbol{\sigma}_T$, vector of thermal stresses in the coordinate system of the shell; c_{ij}' (i, j = 1, 2), coefficients of elasticity determined in the coordinate system of the layer; $\mathbf{C}_\varepsilon'$ and $\mathbf{C}_\gamma'$, matrices of the coefficients of elasticity determined in the coordinate system of the layer; β_ε and β_γ, transformation matrices of the components of the column vector of strains in passage from the coordinate system of the layer to the coordinate system of the shell; $\lambda_{[i]}$, thermal conductivity of the ith layer; φ, angle of reinforcement; T, temperature; H_Σ, thickness of the multiple sandwich; H_0, thickness of the honeycomb filler; h_0, thickness of the casing; k_i (i = 1, 2), curvature of the sandwich; $\sigma_{1T}^{[i]}$ (1, 2), thermal stresses in the ith layer; the notation (1, 2) after formulas means that the numerical indices change from 1 to 2 in the properties of cyclic permutation; $c_{kl}^{[i]}$ (k, l = 1, 2), coeffficients of elasticity of the ith layer; $\alpha_1^{0[i]}$ and $\alpha_2^{0[i]}$, coefficients of linear temperature expansions of the ith layer; $\varphi_{[i]}$, angle of reinforcement of the ith layer; $T_{(i-1)}$, temperature of the interior surface of the ith layer; T_d and T_u, temperature of the lower and upper surface of the multiple sandwich; $z_{(i-1)}$ and $z_{(i)}$, coordinates of the interior and exterior surface of the ith layer relative to the coordinate plane of the sandwich; $h_{[i]}$, thickness of the ith layer; n, number of layers; N_{1T} and N_{2T}, temperature forces in the multiple sandwich per unit length; M_{1T} and M_{2T}, temperature moments in the multiple sandwich per unit length.

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